End of Unit 1 Corrective Assignment – Limits and Continuity

Give the value of each statement. If the value does not exist, write "*does not exist*" or "*undefined*."

1.

a. $\lim_{x\to 3^-} f(x) =$

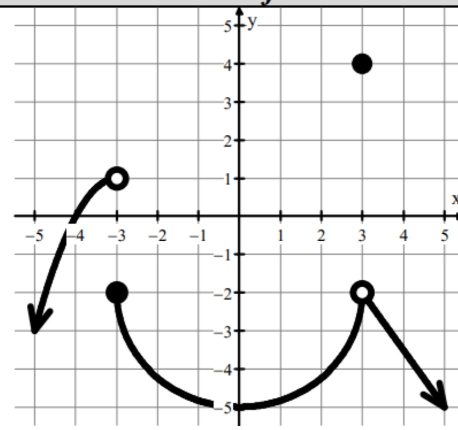
b. $f(-3) =$

c. $\lim_{x\to -3^-} f(x) =$

d. $\lim_{x\to -3^+} f(x) =$

e. $f(3) =$

f. $\lim_{x\to 3} f(x) =$



Evaluate the limit.

2. $\lim_{x\to 1} \frac{\sqrt{x+15}-4}{x-1}$	3. $\lim_{x\to -7^-} \frac{x}{x+7}$	4. $\lim_{x\to\infty} \sin\left(\frac{\frac{\sqrt{2}}{2}x+\pi x^2}{4x^2-x^3+2}\right)$
5. $\lim_{x\to 3} \frac{x-1}{x^2-6x+9}$	6. $\lim_{x\to 3} \frac{x^2-3x}{x^2-9}$	7. $\lim_{x\to 0} \frac{x}{\frac{1}{x+7}-\frac{1}{7}}$
8. $\lim_{x\to\infty} \frac{5x^4-3x^3-1}{x^3-2x^4}$	9. $\lim_{x\to 0} \frac{\sin^2(2x)}{2x^2}$	10. $\lim_{x\to -7^-} \frac{\lvert x+7\rvert}{x+7}$

11. If $f(x) = \begin{cases} \sin(2x), & x < \frac{\pi}{4} \\ \sin x, & \frac{\pi}{4} \le x \le \pi \\ \cos\left(\frac{x}{2}\right), & x > \pi \end{cases}$

find the following:

a. $\lim_{x \to \frac{\pi}{4}^+} f(x) =$

b. $\lim_{x \to \frac{\pi}{4}^-} f(x) =$

c. $\lim_{x \to \pi^-} f(x) =$

d. $\lim_{x \to \pi^+} f(x) =$

e. $f\left(\frac{\pi}{4}\right) =$

f. $f(\pi) =$

g. $\lim_{x \to \frac{\pi}{4}} f(x) =$

h. $\lim_{x \to \pi} f(x) =$

Identify any horizontal asymptote(s) of the following functions		
12. $f(x) = 4^x$	13. $f(x) = \frac{(x+4)(x+1)}{(2x-1)^2}$	14. $f(x) = \frac{\sqrt{9x^4+x^3-4}}{x^2+2x-1}$

15. Let g and h be the functions defined by $g(x) = -x^2 + 2x + 3$ and $h(x) = 2x - 1$. If f is a function that satisfies $g(x) \le f(x) \le h(x)$ for all x, what is $\lim_{x \to -2} f(x)$?

In a certain country, the number of deaths in a year can be modeled by d, where $d(t)$ is the number of deaths and t is the year since 1975 for $0 \le t \le 30$.		
16. What does $d(25)$ represent?	17. What does $\frac{d(30)-d(20)}{30-20}$ represent?	18. What does $\frac{d(15)-d(14.999)}{15-14.999}$ represent?

For each function identify the type of each discontinuity and where it is located.

19. $g(x) = \begin{cases} \ln(ex), & x < 1 \\ 2, & x = 1 \\ x - 1, & 1 < x \leq 2 \\ x^2 - 3 & x > 2 \end{cases}$

20. $f(x) = \frac{x}{x^2-4x}$

21. $f(x) = \frac{x^2+9x+14}{x+7}$

State whether the function is continuous at the given x values. Justify your answers!

22. $f(x) = \begin{cases} \frac{1}{x^2+5}, & x \leq -2 \\ 3^x, & -2 < x < 1 \\ \cos(3\pi x), & x \geq 1 \end{cases}$

a. Continuous at $x = -2$?

b. Continuous at $x = 1$?

Find the domain of each function.

23. $w(t) = \frac{t-7}{\sqrt{t+49}}$

24. $f(x) = \ln(6x - 5)$

25. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2-2x-63}{x-9}$ when $x \neq 9$, then $f(9) =$

26. Let f be the function defined by $f(x) = \begin{cases} \frac{3\sin(5x)}{2x}, & x \neq 0 \\ a, & x = 0 \end{cases}$. For what value of a is f continuous at $x = 0$?

27. According to the table, what is value of $\lim_{x\to 2} f(x)$?

x	1.8	1.99	2.01	2.3
$f(x)$	-5.6	-5.501	-5.499	-5.3

Unit 2 CA – Differentiation: Definition & Fundamental Properties

Find the average rate of change of each function on the given interval. Use appropriate units if necessary.

1. $s(t) = \frac{1}{t-3}$; $[0, 1]$
   s represents miles
   t represents seconds

Use the following table to find the average rate of change on the given interval.

t (item)	2	60	100	200	500
$p(t)$ (dollars)	$-7{,}000$	-100	350	900	2,500

2. $[60, 500]$

3. $2 \le t \le 100$

Each limit represents the instantaneous rate of change of a function. Identify the original function, and the *x*-value of the instantaneous rate of change.

4. $\lim_{x \to 4} \frac{(x^2-3x)-(4)}{x-4}$

   Function: $f(x) =$

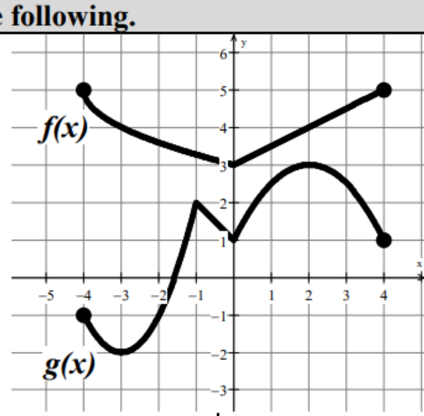
   Instantaneous rate at $x =$

5. $\lim_{h \to 0} \frac{9(5+h)-10(5+h)^2+(205)}{h}$

   Function: $f(x) =$

   Instantaneous rate at $x =$

Use the graphs of f and g to find the following.



6. $h(x) = f(g(x))$. Find the average rate of change on the interval $[2,4]$.

7. $j(x) = g(f(x))$. Find the average rate of change on the interval $[-3,2]$.

Use the table to find the value of the derivatives of each function.

8.

x	$h(x)$	$h'(x)$	$r(x)$	$r'(x)$
-2	-3	2	-2	4

a. $f(x) = -h(x)r(x)$
Find $f'(-2)$.

b. $g(x) = \frac{h(x)+r(x)}{r(x)}$
Find $g'(-2)$.

c. $w(x) = \big(4 - 2h(x)\big)\big(1 - r(x)\big)$
Find $w'(-2)$.

9. $S(x)$ is the number of students in Mr. Kelly's class and x is the number of years since 2015.

a. Explain the meaning of $S(3) = 127$.

b. Explain the meaning of $S'(3) = 4$.

Find the value of the derivative at the given point. Round or truncate to three decimal places.

10. $f(x) = \frac{x}{\sqrt{3x-5}}$ at $x = 5$.

11. $f(x) = \tan^2 x$ at $x = -2$.

12. Use the tables to estimate the value of $f'(45)$. Indicate units of measures.

t minutes	20	30	60	65	75
$v(t)$ feet per minute	100	207	455	501	606

Find the derivative of each function.

13. $s(t) = 7\sin t - 3\ln t - 5e^t$

14. $f(x) = 7x^3 - 4x^2 + x - 3$

15. $f(x) = \frac{x^2}{\cos x}$

16. $y = \sqrt{x} - \cot x$

17. $d(t) = (4 - t)\cos t$

18. $g(x) = \frac{x^4 - 3x^2 + 6x}{x^2}$

19. $g(x) = 4x^3 e^x$

20. $h(x) = 8\sqrt{x} - \frac{5}{x^4} + \pi^2$

21. $g(x) = \frac{3}{4}x^{-1} - \frac{1}{2}\sqrt{x}$

22. At what x-value(s) does the function $f(x) = \frac{x^4}{4} - 3x^3 + 9x^2 + 7$ have a horizontal tangent?

23. If $f(x) = \cos x + \sin x$, find $f'\left(\frac{\pi}{3}\right)$

Find the equation for the tangent line to the function at the given value of x.

24. $f(x) = 6\sqrt{x} + \frac{4}{x} - 1$ at $x = 4$

25. $f(x) = 2e^x + \cos x$ at $x = 0$

26. Is the function differentiable at $x = -1$? $f(x) = \begin{cases} 3x^4 + 9x - 6, & x < -1 \\ \frac{2}{x} - x - 11, & x \geq -1 \end{cases}$

27. What values of a and b would make the function differentiable at $x = 2$?

$$f(x) = \begin{cases} ax^3 + 2x + 1, & x < 2 \\ 2 - bx, & x \geq 2 \end{cases}$$

Unit 3 CA – Composite, Implicit, and Inverse Functions

Find $\frac{dy}{dx}$.

1. $y = \frac{e^{\tan 3x}}{3}$

2. $y = \ln(\sin 5x)$

3. $y = x \ln(4x)$

4. $e^{y^2} = x^5 + 10$

5. $y = \cos^{-1}(7x^3)$

6. $2x^3 - xy = \ln(y)$

Find the equation of the tangent line at the given point.

7. $4x^3 = -5xy + 4y$ at $(1, -4)$

8. $y = \arccos(5x)$ at $x = -\frac{\sqrt{3}}{10}$

9. $h(x) = (2x - 1)^3(x + 2)$ at $x = 1$.

10. Find the equation of any horizontal tangent lines for the graph of $(y^3 + 1)^2 = x^2 + 4x + 4$.

11. Slope of the tangent line of $g(x) = 4\sin^3 x$ at $x = \frac{\pi}{4}$.

12. Let f and g be differentiable functions where $g(x) = f^{-1}(x)$ for all x. $f(6) = 8$, $f(8) = 2$, $f'(6) = -3$, and $f'(8) = 4$. What is the value of $g'(8)$?

Find $\frac{d^2y}{dx^2}$ based on the given information.

13. $y = e^{x^4}$

14. $5y^2 + 3 = x^2$

Evaluate the 2^{nd} derivative at the given point.

15. If $f(x) = x^3 + \frac{5}{x}$, find $f''(-1)$.

16. If $x^2 + y^2 = 13$, find $\frac{d^2y}{dx^2}$ at $(2, 3)$.

The table below gives values of the differentiable functions g and h, as well as their derivatives, g' and h', at selected values of x.

x	$g(x)$	$g'(x)$	$h(x)$	$h'(x)$
-1	0	4	3	6
0	9	2	0	-4
3	-1	-2	9	4
9	3	1	16	9

17. If $f(x) = \frac{g(x)}{\sqrt{h(x)}}$, find $f'(3)$.

18. Find $\frac{d}{dx}h^{-1}(9)$.

19. Find the equation of the tangent line to $g^{-1}(x)$ at $x = 3$.